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# Importance Sampling for Large Sums of i.i.d Random Variables

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## Abstract

We consider the probability that the sum of nonnegative i.i.d random variables falls below a given threshold. We are particularly interested in the rare event regime corresponding to large sums and/or small threshold. Exponential twisting is a popular importance sampling (IS) technique that, in most cases, compares favorably to existing estimators. However, it has several limitations such as sampling under the new measure is not straightforward and might be expensive. We propose an alternative change of measure that yields at least the same performance as the exponential twisting technique and, at the same time, does not introduce serious limitations. For distributions whose probability density functions (PDFs) tend to zero polynomially, we prove that the Gamma IS PDF with appropriately chosen parameters retrieves asymptotically, in the rare event regime, the same performance of the estimator based on the use of the exponential twisting technique. Moreover, in the Log-normal setting, we numerically show that a Gamma IS PDF with optimized parameters clearly outperforms the exponential twisting change of measure. Numerical experiments validate the efficiency of the proposed estimator in delivering a highly accurate estimate in the regime of large sums and/or small threshold.

**Keywords:** rare event regime, importance sampling, large sums, exponential twisting

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