Estimation of the tail-index and extreme quantiles from a mixture of heavy-tailed distributions

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Abstract

The estimation of extreme quantiles requires adapted methods to extrapolate beyond the largest observation of the sample. Extreme-value theory provides a mathematical framework to tackle this problem together with statistical procedures based on the estimation of the so-called tail-index describing the distribution tail. We focus on heavy-tailed distributions and consider the case where the shape of the distribution tail depends on unknown auxiliary variables. As a consequence, one has to deal with observations from a mixture of heavy-tailed distributions, and it is shown that, in such a situation, usual extreme-value estimators suffer from a strong bias. We propose several methods to mitigate this bias. Their asymptotic properties are established and their finite sample performance is illustrated both on simulated and real financial data This is joint work with Emmanuel Gobet.

Keywords: Extreme, values, heavy, tailed distribution, mixture.

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