Regenerative-Simulation-Based Estimators of Risk Measures for Hitting Times to Rarely Visited Sets

Peter W. Glynn , Marvin K. Nakayama , and Bruno Tuffin*[1]

[1]Inria – INRIA – France

Abstract

Consider the hitting time T to a rarely visited set A of a regenerative process $X =$ *[X(t) : t> =0]*. For example, if X represents the queue-length process in a GI/GI/1 queue, T may be the first time the queue length reaches a high level K. For a highly reliable Markovian system comprising a collection of components with exponentially distributed failure and repair times, T can be the first time the system fails.

Let F be the cumulative distribution function (CDF) of T, and we want to estimate F, along with its q-quantile and conditional tail expectation (CTE). In various asymptotic settings, the distribution of *T/E[T]* converges to an exponential as the set A becomes rarer. Thus, we can approximate F by an exponential with mean *E[T]*. As the mean *E[T]* is unknown, we estimate it via simulation with measure-specific importance sampling to calibrate the approximation. This leads to our so-called exponential estimators of F and the corresponding risk measures.

Moreover, as X is regenerative, we can write $T = S + V$, where S is a geometric sum of lengths of cycles before the one that hits A and V is the time to hit A in the first cycle that visits A. In various asymptotic settings, we also have that *S/E[S]* converges weakly to an exponential. As S and V are independent by the regenerative property, we can then write the CDF F of T as the convolution of the CDF G of S and the CDF H of V. We then exploit this to construct so-called convolution estimators of F and its corresponding risk measures, where we replace G with an exponential with mean *E[S]*, and we estimate H and *E[S]* via simulation with measure-specific importance sampling.

During this talk, we examine the behavior of the exponential and convolution estimators. Through simple models, we show that the weak convergence to an exponential may hold for *S/E[S]* but not for *T/E[T]*. Thus, the convolution estimator may be valid, but the exponential estimator may not be.
We also discuss the bias of estimators. Indeed, for moderately rare events, bias could potentially surpass variance. For this reason, we propose other estimators, still based on the same regenerative principle, and compare all of them numerically.

This is joint work with Peter W. Glynn and Marvin. K. Nakayama.

Keywords: Rare events, regenerative simulation

*Speaker