Splitting for a non-Markovian tandem queue

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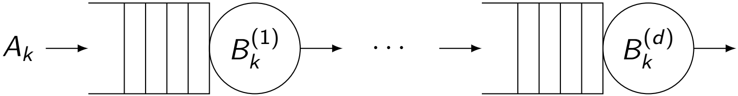
Introduction



- d queues in tandem
- A_k - inter-arrival time of customer $k+1$
- $B_k^{(j)}$ - service time of customer k in queue j
- Interested in the probability that the total number of customers reaches N before the system is empty again $= p_N$
- For large N, this is a rare event when the system is stable.

Introduction - Splitting

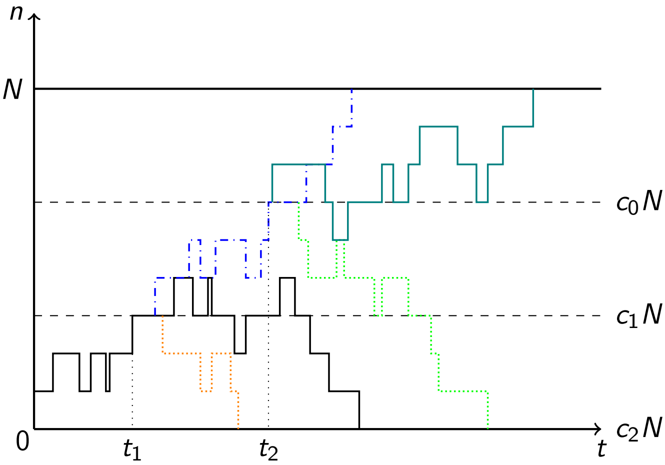


Figure: An example of splitting: a possible realization of particles and splitting thresholds. In this example, $\hat{p}_N = \frac{2}{9}$.

Introduction

- We want the estimator of p_N to be *asymptotically efficient*.
- This means that the relative error grows less than exponentially fast in N *and* that the computational effort grows less than exponentially fast in N.

Introduction

For the same model, importance sampling has been shown to be asymptotically efficient under some conditions.

For splitting, similar conditions turn out *not* to be necessary.

State space

Let $\mathbf{Z}_j = (Z_{1,j}, \ldots, Z_{d,j}, \bar{Z}_{0,j}, \ldots, \bar{Z}_{d,j})$ be the state after j transitions.

- $Z_{i,j}$ is the number of customers at queue i
- $\bar{Z}_{0,j}$ is the residual inter-arrival time
- $\bar{Z}_{i,j}$ is the residual service time at queue i

We start with a busy cycle, i.e. $\mathbf{Z}_0 = (1, 0, \ldots, 0, 0, \ldots, 0)$

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$$\mathbb{V}_Z(\mathbf{z}) = \begin{cases} \{(1, -\bar{z}_0 + a, -\bar{z}_0) : a \geq 0\} & \text{if } \bar{z}_0 < \bar{z}_1 \\ \{(-1, -\bar{z}_1, -\bar{z}_1 + \mathbb{1}\{z_1 > 1\} b_1) : b_1 \geq 0\} & \text{if } \bar{z}_0 \geq \bar{z}_1 \\ \{(0, a, b_1) : a, b_1 \geq 0\} & \text{if } \mathbf{z} = \mathbf{Z}_0, \end{cases}$$

where a, b_1 are any realization of the random variables A and $B^{(1)}$ respectively.

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This means that depending on the state it is known which type of transition to take and almost each of them has infinitely many possibilities.

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Let $\mathbf{X}_j = \frac{\mathbf{Z}_j}{N}$ be the *scaled* state of the system.

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- Then $U(\mathbf{x}) = \min\{\gamma(\mathbf{0}), \gamma(\mathbf{0}) - \gamma(\mathbf{x})\} = \gamma(\mathbf{0}) - \gamma(\mathbf{x})$.

The (negative) decay rate

$$
\begin{aligned}
-\gamma(\mathbf{x}) &= \lim_{N\to\infty} \frac{1}{N} \log \mathbb{P}(K_N(\mathbf{x}_N) < K_0(\mathbf{x}_N)) \\
&= \begin{cases}
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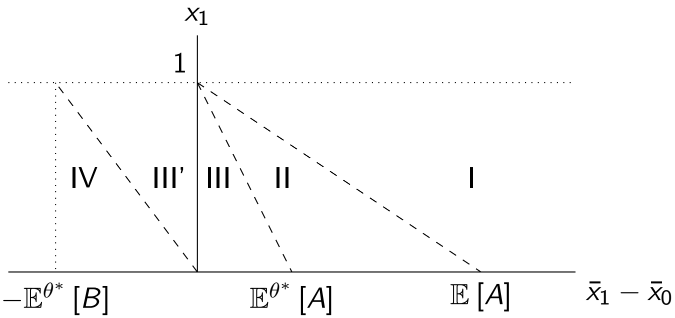

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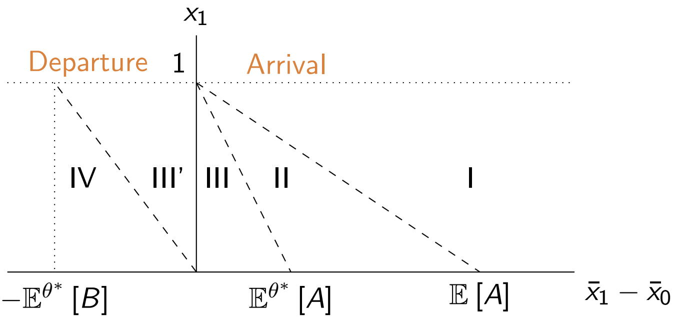

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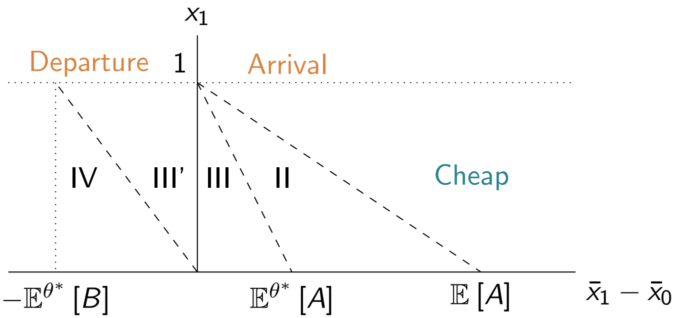

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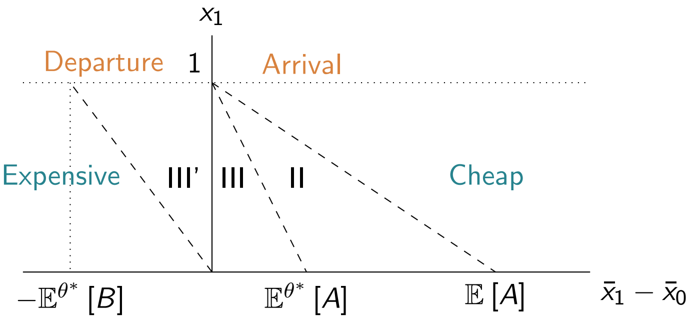

The (negative) decay rate - sketch of the proof

Sketch of the proof for the upper bound:

$$\mathbb{P}\left(K_N(\mathbf{x}_N) < K_0(\mathbf{x}_N)\right) = \mathbb{E}^{\theta,\tilde{\theta}}\left[L^{\theta,\tilde{\theta}} \mid K_N(\mathbf{x}_N) < K_0(\mathbf{x}_N)\right]\mathbb{P}^{\theta,\tilde{\theta}}\left(K_N(\mathbf{x}_N) < K_0(\mathbf{x}_N)\right)$$

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Sketch of the proof for the lower bound:

- Similar proof as Sadowsky 1991 when $\bar{x}_0 = \bar{x}_1$.
- Lower bound probability by the product of the probability to end up in a state where $\bar{x}_0 = \bar{x}_1$ and the probability to reach the overflow level starting from this new state.

Proof of asymptotic efficiency

Need to show:

$$\limsup_{N\to\infty} \frac{1}{N} \log \left(w(N) R^{-2J_N} \mathbb{E}\left[T^2\right] \right) \leq -2\gamma(\mathbf{0}),$$

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- The result follows.

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- Proofs for asymptotic efficiency are extendable to d-nodes.

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Thank you for your attention!

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