A reformulated chance constraint optimization problem for the fatigue design of an offshore wind turbine mooring system

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Context

- What is a **Floating Offshore Wind Turbine** (FOWT)?

- Advantages:
  - Extend the operating sites (water-depth > 50m);
  - Higher average wind speed;
  - More steady wind;
  - Facilitate social acceptance.

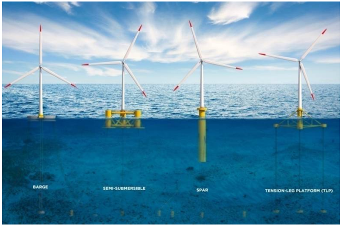

Figure: Different designs of FOWT. Source : [1].

- Need for a methodology to build **affordable** and **reliable** structures.
  - Need to reduced the Levelized Cost of Energy (200€/MWh currently) for pre-commercial projects [1].

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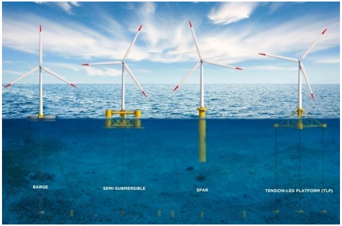

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Focus on the mooring system

- Mooring system:
  - 3 catenary mooring lines composed of chains and connected along the inner cylinder of outer columns.
  - Significant contribution to capital cost;
  - Essential to ensure station-keeping under wind and wave loading;
  - Need to limit Ultimate and Fatigue Limit State (estimation challenging due to the large number of simulations needed).

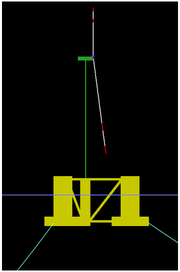

Figure: FOWT's profile

- Case study inspired by NREL 5MW turbine on DeepCWind floater [2];
- Simplifications:
  - **no consideration of second order wave forces**;
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  - **the wind speed is constant on each interval of stationarity**.

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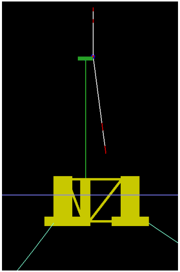

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Minimization of the cost of the mooring system

Problem : **optimize the cost** of the mooring system under reliability constraints.

- Objective function:
  - **cost** of the mooring system (deterministic).
- Design variables:
  - **Additional length to the mooring lines** (in m);
  - **Lineic mass** of the mooring lines (in kg/m);
  - **Position of the connection of the lines to the columns**: from the bottom (0) to the top (1) of the column.

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Constraints and uncertainties

- Constraints (outputs of a numerical simulator - Deeplines™):
  - **Surge** must stay below a threshold of 10 meters (temporary overconservative threshold to illustrate the methodology on the case study);
  - **Tension** of the lines must stay positive;
  - **Fatigue** (accumulated damage) must stay below a resistance threshold R.

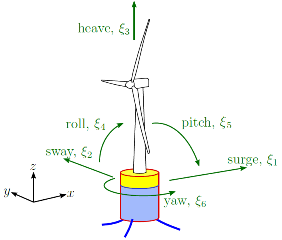

Figure: Degrees of freedom.

- Uncertainties are considered on:
  - the **model** (azimuth, damping and fatigue law coefficients) and on the **resistance** represented by a r.v. X_p;
  - the **sea elevation** represented by a stochastic process $Z_{X_{LT}}$.

Figure: Out of plane bending.

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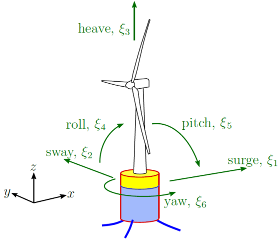

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Figure: Out of plane bending.

Some details and simplifications of our problem

- X_{LT} is a sequence of random i.i.d triplets

$$X_{LT} = ((H_s, T_p, U)_i\,,\ i = 1, \dots, n)$$

  with known joint probability law : 7 possible outcomes (h_s^j, t_p^j, u^j) with probability p^j $(j = 1, \dots, 7)$.

- At $X_{LT} = x_{LT}$, the sea elevation $Z_{x_{LT}}$ is a piecewise stationary process :

$$Z_{x_{LT}}(t) = \sum_{i=1}^{n} Z_{(h_s,t_p,u)_i}(t)\mathbb{1}_{I_i}(t)$$

  For each time interval I_i:
  - $Z_{(h_s,t_p,u)_i}(t)$ is a **stationary Gaussian process** defined by its **spectral density** (JONSWAP [4]) parameterized by $(h_s, t_p, u)_i$.

- we consider the $Surge$ and $Tension$ processes as outputs of linear filters.

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Mathematical formulation

Time-variant Reliability-Based Design Optimization (RBDO) :

$$\min_{d \in \Omega_d} \quad cost(d)$$

$$\text{s.t} \quad \mathbb{P}_{X_p;Z_{X_{LT}}}\left(\max_{[0,T]} \text{Surge}_{d,X_p;Z_{X_{LT}}}(t) > 10\right) < 10^{-4}$$

$$\mathbb{P}_{X_p;Z_{X_{LT}}}\left(\min_{[0,T]} \text{Tension}_{d,X_p;Z_{X_{LT}}}(t) < 0\right) < 10^{-4}$$

$$\mathbb{P}_{X_p;Z_{X_{LT}}}\left(\text{Fatigue}_{[0,T]}(d, X_p; Z_{X_{LT}}) > R\right) < 10^{-4}$$

We consider $T = 1$ year.

Difficulty

Estimation of the failure probabilities at each iteration of the optimization algorithm.

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At $X_p = x_p$ fixed and with Z stationary:

- $Surge_{d,x_p;Z}$ output of **linear filter**:

$$Surge_{d,x_p;Z}(t) = h_{d,x_p} * Z(t)$$

- Z Gaussian, stationary with known psd (JONSWAP)

$\Rightarrow Surge_{d,x_p;Z}$ is also **Gaussian** and **stationary**;

$\Rightarrow$ the mean and spectral moments of $Surge_{d,x_p;Z}$ are computable from the **mean**, **spectral density** of Z and the **transfer function** $FT(h_{d,x_p})$ (given by Deeplines^{TM}).

The same reasoning applies to the $Tension_{d,x_p;Z}$ process.

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Extreme value theory

Theorem (5)

Let ζ be a standardized stationary Gaussian process with spectral density K_ζ. Then:

$$\mathbb{P}\left(a_T\left(\max_{t\in[0,T]}\zeta(t)-b_T\right)\leqslant\alpha\right)\to\exp(-e^{-\alpha})\ \text{ as }\ T\to\infty$$

where

$$a_T=\sqrt{2\log(T)}\ ,\ b_T=a_T+\frac{\log(\frac{\sqrt{\lambda_2}}{2\pi})}{a_T}\ \text{ and }\ \lambda_2=\frac{1}{2\pi}\int_{\mathbb{R}}\nu^2K_\zeta(\nu)d\nu.$$

Thus, for T large enough:

$$\mathbb{P}\left(\max_{[0,T]}\zeta(t)\leqslant\alpha\right)\simeq\exp\left(-e^{a_T(b_T-\alpha)}\right).$$

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Application of the theorem to extreme value constraints

$$\mathbb{P}_{X_p;Z}\left(\max_{[0,T]} \mathrm{Surge}_{d,X_p;Z}(t) > 10\right)$$
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At X_p fixed, $Surge_{d,x_p;Z}$ is a stationary Gaussian process.

$\Rightarrow$ for T large enough, we have:

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where a_T depends only on T and b_T depends on T and on the second spectral moment of the process $Surge_{d,x_p;Z}$.

The same reasoning applies to the constraint on the lines tension.

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Fatigue constraint reformulation with Dirlik

$$\mathbb{P}_{X_p;Z}\Big(\text{Fatigue}_{[0,T]}(d, X_p; Z) > R\Big) < 10^{-4}$$

- Usual approach:
  1. computation of time series of tension;
  2. cycle counting method + Miner's rule + material fatigue law;
  3. $\text{Fatigue}_{[0,T]}(d, x_p, Z) \in \mathbb{R}^+$ (failure if $> R$).

- Dirlik approach [5]: empirical formula to estimate the distribution of the amplitudes of tension cycles from the spectral moments of the tension process.

$$\mathbb{E}_Z\Big[\text{Fatigue}_{[0,T]}(d, x_p, Z)\Big] \simeq \text{Fatigue}^{Dir}_{[0,T]}(d, x_p)$$

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Why the reformulation makes resolution easier

For each d and x_p, it requires computation of **several time series realizations** of Surge and $\mathrm{Tension}$ processes to estimate these probabilities.

$$\mathbb{P}_{X_p;Z}\left(\max_{[0,T]} \mathrm{Surge}_{d,X_p;Z}(t) > 10\right) \qquad \mathbb{P}_{X_p;Z}\left(\mathrm{Fatigue}_{[0,T]}(d, X_p; Z) > R\right)$$

↓ ↓

For each d and x_p, we only need Surge and $\mathrm{Tension}$ **means and spectral moments** given by the psd of Z and the transfer functions.

$$\mathbb{E}_{X_p}\left[1 - \exp\left(-e^{a_T\left(b_T(d,X_d) + \frac{\mu_S(d,X_d)-10}{\sigma_S(d,X_p)}\right)}\right)\right]$$

$$\mathbb{E}_{X_p}\left[\Phi\left(\frac{\ln\left(\mathrm{Fatigue}^{Dir}_{[0,T]}(d,X_p)\right) - c_1(d_2,\mu_R)}{c_2(d_2,\mu_R)}\right)\right]$$

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Stationary problem reformulation

$$\min_{d \in \Omega_d} \quad cost(d)$$

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with b_T, c_T, μ_S, μ_T, σ_S, σ_T, $Fatigue^{Dir}_{[0,T]}$ only depend on the transfer function (which depend on d and x_p) and on the spectral density on the process Z.

From stationary to piecewise stationary reformulation

$$\min_{d\in\Omega_d} cost(d) \qquad \text{such that}$$

$$\mathbb{E}_{X_p}\left[1-\exp\left(-\sum_{j=1}^{7} e^{a_{Tp^j}\left(b_{Tp^j}(d,X_p,h_s^j,t_p^j,u^j)+\frac{\mu_S(d,X_p,h_s^j,t_p^j,u^j)-10}{\sigma_S(d,X_p,h_s^j,t_p^j,u^j)}\right)}\right)\right] < 10^{-4}$$

$$\mathbb{E}_{X_p}\left[1-\exp\left(-\sum_{j=1}^{7} e^{a_{Tp^j}\left(c_{Tp^j}(d,X_p,h_s^j,t_p^j,u^j)-\frac{\mu_T(d,X_p,h_s^j,t_p^j,u^j)}{\sigma_T(d,X_p,h_s^j,t_p^j,u^j)}\right)}\right)\right] < 10^{-4}$$

$$\mathbb{E}_{X_p}\left[\Phi\left(\frac{\ln\left(T\sum_j p^j \text{Fatigue}_{[0,T]}^{Dir}(d,X_p,h_s^j,t_p^j,u^j)\right)-c_1(d_2,\mu_R)}{c_2(d_2,\mu_R)}\right)\right] < 10^{-4}$$

- New method : Adaptive Kriging for Expectation Constraint Optimization (**AK-ECO**).

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Metamodeling strategy

Surge constraint :

$$\mathbb{E}_{X_p}\left[1-\exp\left(-\sum_{j=1}^{7} e^{a_{T_p^j}\left(b_{T_p^j}(d,X_p,h_s^j,t_p^j,u^j)+\frac{\mu_{\mathrm{S}}(d,X_p,h_s^j,t_p^j,u^j)-10}{\sigma_{\mathrm{S}}(d,X_p,h_s^j,t_p^j,u^j)}\right)}\right)\right]$$

$$\mathbb{E}_{X_p}\left[1-\exp\left(-\sum_{j=1}^{7}\exp\left(M(d,X_p,h_s^j,t_p^j,u^j)\right)\right)\right]$$

From a space-filling design of experiments (DoE), calibration of a metamodel by **Gaussian process regression** or **Kriging**:

$$\widetilde{M}(d,x_p,h_s,t_p,u)\sim\mathcal{N}\left(\mu_M(d,x_p,h_s,t_p,u),\sigma_M(d,x_p,h_s,t_p,u)^2\right)$$

- Idea of AK-ECO : succession of cycles composed of a local enrichment of metamodels and optimization resolution.

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Solving the RBDO problem: AK-ECO

- **Step 1:** Initial design d^0, initial metamodels and cycle $= 1$.
- **Step 2:** local enrichment of metamodels:
  - If $\widetilde{M}^{cycle-1}$ accurate enough around $d^{cycle-1}$
  - Else: sequential local enrichment of $\widetilde{M}^{cycle-1}$ around $d^{cycle-1}$.

  $\widetilde{M}^{cycle-1} \longrightarrow \widetilde{M}^{cycle}$.
- **Step 3:** solve the RBDO from design $d^{cycle-1}$.
  - optimization algorithm: Sequential Quadratic Approximation [6]
  - constraints evaluated through Monte Carlo applied on $\widetilde{M}^{cycle}$.

  Retrieve d^{cycle}.
- **Step 4:**
  - If $cycle > 1$, $||d^{cycle} - d^{cycle-1}|| < \epsilon$ and $\widetilde{M}^{cycle}$ accurate enough around d^{cycle}: the final minimum is d^{cycle}.
  - Else: $cycle = cycle + 1$ and go back to step 2.

[6] H. Langouët, *Optimisation sans dérivées sous contraintes : deux applications industrielles en ingénierie de réservoir et en calibration des moteurs.* PhD thesis, Université Nice Sophia Antipolis, 2011.

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Focus on step 2 : local enrichment

$$\widetilde{M}^{cycle}(d, x_p, h_s, t_p, u) \sim \mathcal{N}\left(\mu^{cycle}(d, x_p, h_s, t_p, u), \sigma^{cycle}(d, x_p, h_s, t_p, u)^2\right)$$

Estimation of the surge constraint at d^{cycle}:

$$\frac{1}{N_{MC}} \sum_{i=1}^{N_{MC}} 1 - \exp\left(-\sum_{j=1}^{7} \exp\left(\mu^{cycle}\left(d^{cycle}, x_p^i, h_s^j, t_p^j, u^j\right)\right)\right) \quad (1)$$

Procedure of local enrichment at d^{cycle}:

1. select x_p^i $(i \in \{1, \ldots, N_{MC}\})$ and h_s^j, t_p^j, u^j $(j = 1, \ldots, 7)$ that maximizes a **new criterion** specific to AK-ECO. This criterion favors points where the uncertainty of prediction σ^{cycle} implies big uncertainties on the MC estimation (1);
2. evaluate the response at this point (one code call);
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Comparaison of AK-ECO with reference methods in RBDO

Results of the different methods with $d^0 =$ (-1.07, 138.9, 0.07):

	MC+KM1600	SORA[7]	Stieng[8]	AK-ECO
d^{min}	$(1.11, 110.45, 0)$	$(-0.2, 101.2, 0)$	$(-1.99, 86.6, 0)$	$(1.05, 109.96, 0)$
$cost(d^{min})$	0.262	0.201	0.104	0.259
$p_{\mathcal{S}}(d^{min})$	1.0×10^{-4}	0.9×10^{-4}	0.66	1.0×10^{-4}
$p_{\mathcal{F}}^{line_3}(d^{min})$	1.0×10^{-4}	2.6×10^{-4}	14.2×10^{-4}	1.0×10^{-4}
N_{calls}	1600	24983	4557	456

- AK-ECO finds a reliable optimum with few calls to the expensive code.

[7] X. Du and W. Chen, *Sequential Optimization and Reliability Assessment Method for Efficient Probabilistic Design*, Journal of Mechanical Design, 2004

[8] Lars Einar S. Stieng. *Optimal design of offshore wind turbine support structures under uncertainty*. PhD thesis, Norwegian University of Science and Technology, 2019.

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Conclusion

- **Optimization problem** with **probabilistic constraints** dependent on a **random vector** and a **piecewise stationary stochastic Gaussian process**.
- Methodology in two parts:
  - Reformulate the problem thanks to **extreme value theory** and the **Dirlik** approach;
  - Solve the reformulated problem with a **new adaptive kriging** strategy **AK-ECO** and a **new learning criterion**.
- Successful application of the methodology to the academic case of a harmonic oscillator and to the wind turbine case.
- Perspective: quantify the approximation errors made during the reformulation steps.

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Appendix : enrichment criterion

$$\max_{i\in\{1,\dots,N_{MC}\},j\in\{1,\dots,7\}} \mathcal{C}_{aug}(d, x_p^i, h_s^j, t_p^j, u^j) \quad \text{where} \quad \mathcal{C}_{aug}(d, x_p^i, h_s^j, t_p^j, u^j)$$

$$= \Bigg[F_5\Bigg(\sum_{j'\neq j} \exp\Big(\mu(d, x_p^i, h_s^{j'}, t_p^{j'}, u^{j'})\Big) + \exp\Big(\Big(\mu(d, x_p^i, h_s^j, t_p^j, u^j)\Big) + 2\sigma(d, x_p^i, h_s^j, t_p^j, u^j)\Big) \Bigg)$$

$$- F_5\Bigg(\sum_{j'\neq j} \exp\Big(\mu(d, x_p^i, h_s^{j'}, t_p^{j'}, u^{j'})\Big) + \exp\Big(\Big(\mu(d, x_p^i, h_s^j, t_p^j, u^j)\Big) - 2\sigma(d, x_p^i, h_s^j, t_p^j, u^j)\Big) \Bigg) \Bigg]$$

$$\times f_{X_p}(x_p^i)$$

$$F_5(x) = 1 - \exp(-x)$$