# CLIMATE EXTREME EVENT ATTRIBUTION and MULTIVARIATE EXTREME VALUE THEORY

LSCE

Laboratoire des sciences du climat & de l'environnement

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Joint work with Anna Kiriliouk, Julien Worms, Soulivanh Thao, Alexis Hannart and Aurélien Ribes.

# Outline

## Climate models and Detection & Attribution (D&A )

- Concepts

## Extremes and attribution

- Multivariate extreme value theory

# The statistical setup

**Return level $u_T$ for the time period $T$**

$$\mathbb{P}(X > u_T) = \frac{1}{T}$$

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## Return period for a weighted sum

$$\mathbb{P}(\omega_1 X_1 + \omega_2 X_2 > u_T) = \frac{1}{?}$$

with $\mathbb{P}(X_1 > u_T) = \mathbb{P}(X_2 > u_T) = \frac{1}{T}$

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with $X_i$ unit exponentials and $u_T = \log T$

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| | $w_1 = 0.2,\ w_2 = 0.8$ | | |
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| Independence | 13 | 100 | 237 |

**Learned lesson**

Given identically distributed univariate variables with the same return level $u_T$, the **degree of dependence** in the original data and the **weights** greatly influence the return period of the event $\{\omega_1 X_1 + \omega_2 X_2 > u_T\}$

## The climatological setup

## Attribution

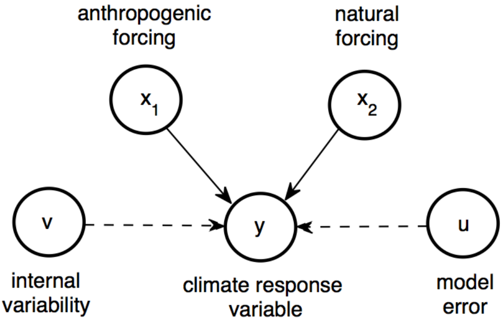
Evaluating the relative contributions of multiple causal factors [1] to a change or event with an assignment of statistical confidence.

## Consequences

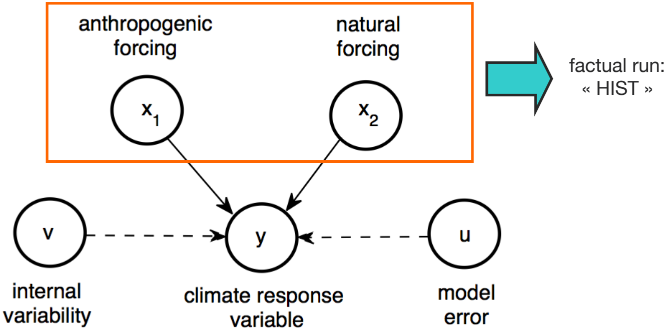
Need to assess whether the observed changes are

- consistent with the expected responses to external forcings (PS)
- inconsistent with alternative explanations (PN)

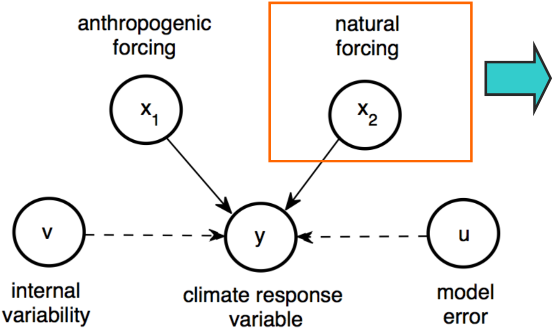
1. casual factors usually refer to external influences, which may be anthropogenic (GHGs, aerosols, ozone precursors, land use) and/or natural (volcanic eruptions, solar cycle modulations


anthropogenic forcing
natural forcing
x1
x2
v
y
u
internal variability
climate response variable
model error

# Factual world

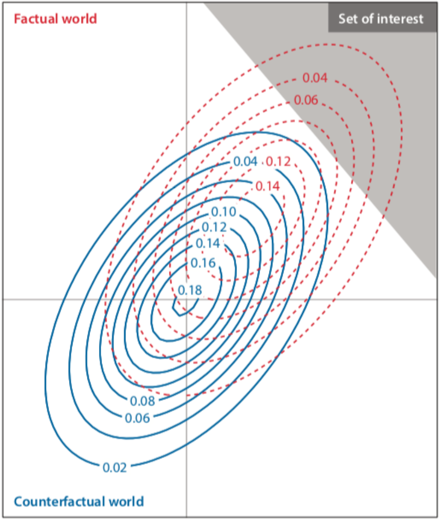


# Counterfactual world




counterfactual run
w.r.t.
anthropogenic forcing:
« NAT »

# Counterfactual and factual world in a probabilistic framework



# The so-called event attribution scheme

## Fraction of Attributable Risk (FAR)

Relative ratio of two probabilities, $p_0$ the probability of exceeding a threshold in a “world that might have been (no antropogenic forcings)" and $p_1$ the probability of exceeding the same threshold in a “world that it is"

$$FAR = 1 - \frac{p_0}{p_1}.$$

(see Stott P. A., Stone D. A., Allen M. R. (2004). Human contribution to the European heatwave of 2003. Nature)

# FAR is linked with Pearl counter-factual theory

Hannart, Pearl, Otto, PN and Ghil. Counterfactual causality theory for the attribution of weather and climate-related events, BAMS, 2015

PN, Hannart and Ribes, Statistical Methods for Extreme Event Attribution in Climate Science, Annual Rev. of Stat. and Its Appli., 2020

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## Causality cheat sheet

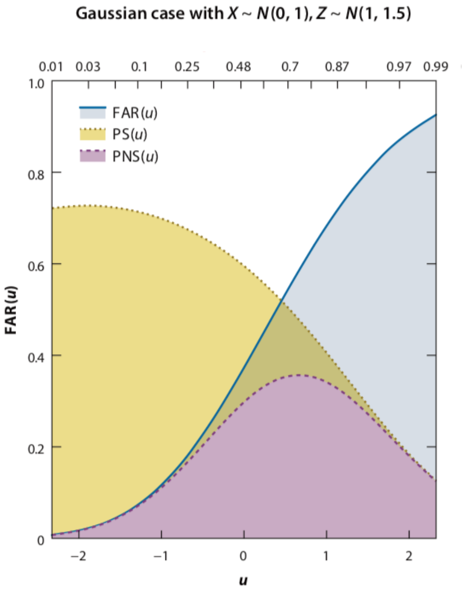
$$\text{Necessary causation} = \text{PN} = FAR = \max\left(1 - \frac{p_0}{p_1}, 0\right),$$

$$\text{Sufficient causation} = \text{PS} = \max\left(1 - \frac{1 - p_1}{1 - p_0}, 0\right),$$

$$\text{Both causation} = \text{PNS} = \max\left(p_1 - p_0, 0\right),$$

where $p_0$ proba in the counterfactual world & $p_1$ in the factual one

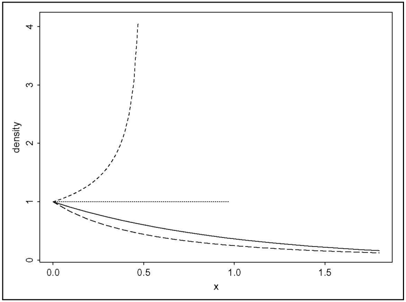
## Gaussian example with $p_0 = P(X > u)$ and $p_1 = P(Z > u)$



# Another example : the Generalized Pareto Distribution (GP) survival function

$$\overline{H}_\gamma(x/\sigma) = \left(1 + \frac{\gamma\, x}{\sigma}\right)_{+}^{-1/\gamma}$$

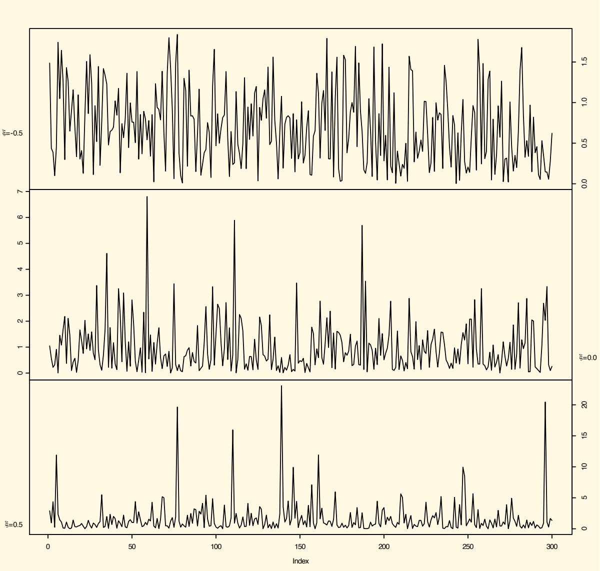
Vilfredo Pareto : 1848-1923



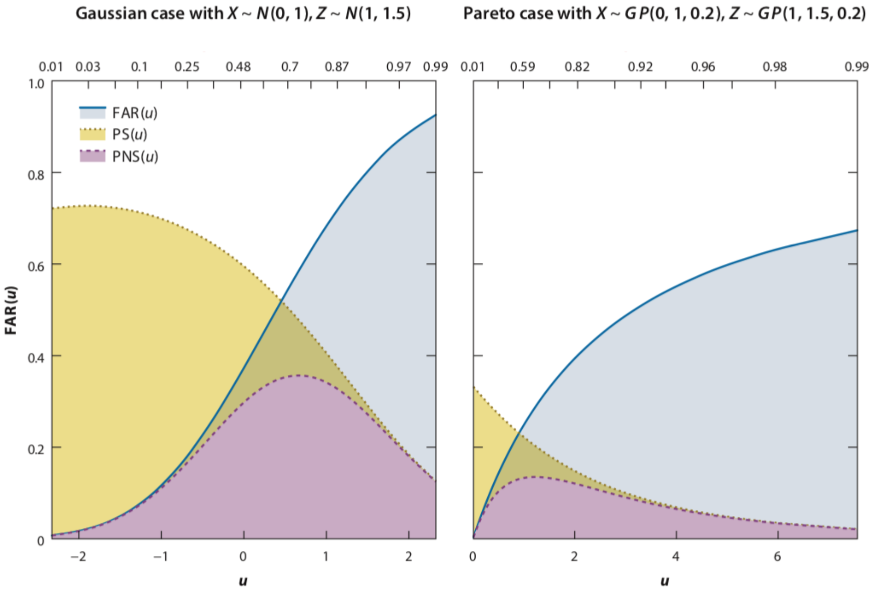

Born in France and trained as an engineer in Italy, he turned to the social sciences and ended his career in Switzerland. He formulated the power-law distribution (or "Pareto's Law"), as a model for how income or wealth is distributed across society.

see, e.g. *Statistics of Extremes A.C. Davison and R. Huser Annual Review of Statistics and Its Application 2015 2 :1, 203-235*

# From Bounded ($\gamma < 0$) to Heavy tails ($\gamma > 0$)



## Examples with $p_0 = P(X > u)$ and $p_1 = P(Z > u)$



Moving to multivariate extremes

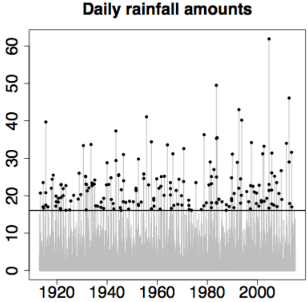
*The Annals of Applied Statistics*
2020, Vol. 14, No. 3, 1342–1358
https://doi.org/10.1214/20-AOAS1355


# CLIMATE EXTREME EVENT ATTRIBUTION USING MULTIVARIATE PEAKS-OVER-THRESHOLDS MODELING AND COUNTERFACTUAL THEORY

BY ANNA KIRILIOUK[1] AND PHILIPPE NAVEAU[2]

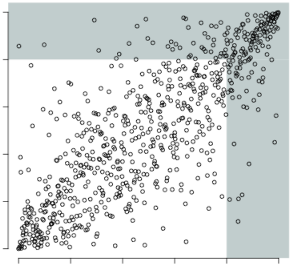
**Univariate modelling strategy:**

- Fix some high threshold $u$.
- Fit a GPD to the conditional threshold excesses $Y - u \mid Y > u$.




**Multivariate modelling strategy:**

- Fix some high threshold $\boldsymbol{u}$.
- Fit a multivariate GPD to the conditional threshold excesses $\boldsymbol{Y} - \boldsymbol{u} \mid \boldsymbol{Y} \nleq \boldsymbol{u}$.



## Multivariate generalized Pareto definition

### Simulation

$$\mathbf{V}_{\gamma=0} \stackrel{\mathrm{d}}{=} E + \mathbf{T} - \max_{1 \le j \le d} T_j,$$

where $E \sim \mathrm{Exp}(1)$ and $\mathbf{T}$ a $d$-dimensional r.v., $\perp E$.

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where $E \sim \mathrm{Exp}(1)$ and $\mathbf{T}$ a $d$-dimensional r.v., $\perp E$.

### Exponential conditional marginals

$$\mathbb{P}[V_i > z | V_i > 0] = \exp(-z)$$

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## Different marginals

Every MGP vector has a representation on $\{\mathbf{v} \in \mathbb{R}^d : \mathbf{v} \not\leq \mathbf{0}\}$ as

$$\mathbf{V} \stackrel{\mathrm{d}}{=} \sigma \frac{e^{\gamma \mathbf{V}_{\gamma=0}} - \mathbf{1}}{\gamma}$$

and

$$\mathbb{P}[V_j > v \mid V_j > 0] = (1 + \gamma_j v / \sigma_j)_+^{-1/\gamma_j},$$

# MGPD simulated samples with density contours

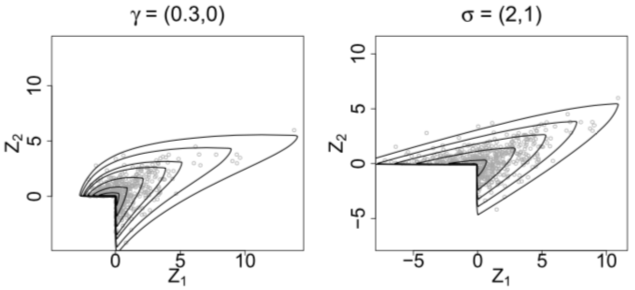


FIG. 1. *Scatterplots and density contours from 500 bivariate GPD random draws using (2*
*parameters* $\boldsymbol{\gamma} = (0.3, 0)$, $\boldsymbol{\sigma} = (1, 1)$ *for the left panel and* $\boldsymbol{\gamma} = (0, 0)$, $\boldsymbol{\sigma} = (2, 1)$ *for the right pa*
$\boldsymbol{T}$ *is zero-mean bivariate Gaussian with unit covariance matrix* $I_2$.

## Theoretical justification

Let $\mathbf{Y} \in \mathbb{R}^d$ and $\mathbf{Y}_i = (Y_{i1}, \ldots, Y_{id})$, $i \in \{1, \ldots, n\}$, $n$ iid copies of $\mathbf{Y}$. Let $\mathbf{M}_n := (M_{n,1}, \ldots, M_{nd})$ with $M_{nj} := \max(Y_{1j}, \ldots, Y_{nj})$

**Max-domain of attraction**

$$\mathbb{P}\left[\frac{\mathbf{M}_n - \mathbf{b}_n}{\mathbf{a}_n} \leq \mathbf{x}\right] = \mathbb{P}^n\left(\mathbf{Y} \leq \mathbf{a}_n\mathbf{x} + \mathbf{b}_n\right) \xrightarrow{\text{d}} MGEV(\mathbf{x}), \qquad \text{as } n \to \infty. \tag{1}$$

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**Multivariate Generalized Pareto (GP) r.v. (Rootzen & Tajvidi, 2006)**

Let $\mathbf{l} = (l_1, \ldots, l_d)$ the lower endpoints vector of $G$. If (1) holds,

$$\max\left\{\frac{\mathbf{Y} - \mathbf{b}_n}{\mathbf{a}_n}, \mathbf{l}\right\} \mid \mathbf{Y} \not\leq \mathbf{b}_n \xrightarrow{d} \mathbf{V}, \qquad \text{as } n \to \infty,$$

where $\mathbf{V}$ is said to follow a multivariate GP distribution with cdf $H$.

# Multivariate generalized Pareto properties

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**Linear projection (Rootzen et al. 2016)**

If $\gamma = \gamma\mathbf{1}$, then for any weights $\mathbf{w} = (w_1, \ldots, w_d) > \mathbf{0}$ such that $\mathbb{P}[\mathbf{w}^T\mathbf{V} > 0] > 0$,

$$\mathbf{w}^T\mathbf{V} \mid \mathbf{w}^T\mathbf{V} > 0 \sim \mathrm{GP}(\mathbf{w}^T\sigma, \gamma).$$

MGPD and maximizing causality

## Causality cheat sheet

$$\begin{aligned} \mathrm{PN} = \mathit{FAR} &= \max\left(1 - \frac{p_0}{p_1}, 0\right), \\ \mathrm{PS} &= \max\left(1 - \frac{1-p_1}{1-p_0}, 0\right), \\ \mathrm{PNS} &= \max\left(p_1 - p_0, 0\right), \end{aligned}$$

where $p_0$ proba in the counterfactual world & $p_1$ in the factual one

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where $p_0$ proba in the counterfactual world & $p_1$ in the factual one

## Optimal projection that maximizes PN

Find weights $\mathbf{w} \in \{\mathbf{w} \in [0,1]^d : w_1 + \cdots + w_d = 1\}$ that maximizes

$$\mathrm{PN}(v, \mathbf{w}) = \max\left(1 - \frac{\mathbb{P}[\mathbf{w}^T \mathbf{X}^{(0)} > v]}{\mathbb{P}[\mathbf{w}^T \mathbf{X}^{(1)} > v]}, 0\right).$$

## Optimal projection that maximizes PN

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## Homegeneous case with $\gamma = \gamma \mathbf{1}_d$

$$\begin{aligned}\mathbb{P}[\mathbf{w}^T \mathbf{X} > v] &= \mathbb{P}[\mathbf{w}^T \mathbf{X} > \mathbf{w}^T \mathbf{u}]\, \mathbb{P}[\mathbf{w}^T (\mathbf{X} - \mathbf{u}) > v - \mathbf{w}^T \mathbf{u} \mid \mathbf{w}^T (\mathbf{X} - \mathbf{u}) > 0] \\ &\approx \mathbb{P}\left[\mathbf{w}^T \mathbf{X} > \mathbf{w}^T \mathbf{u}\right] \overline{H}\left(v - \mathbf{w}^T \mathbf{u}; \mathbf{w}^T \sigma, \gamma\right),\end{aligned}$$

# Necessary gain from univariate to multivariate analysis (equal weights but different dependence strengths)

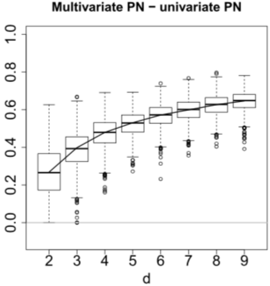



FIG. 6. *Boxplots of the multivariate estimates* $\widehat{\mathrm{PN}} = 1 - \widehat{p}_0/\widehat{p}_1$ *minus the univariate PN estimates of aggregated data, where* $\widehat{p}_i$ *is defined in* (4.4), *and* $d \in \{2, \ldots, 9\}$. 1000 *samples of size* $n = 2000$ *were simulated from a multivariate Gaussian GPD model with* $\boldsymbol{\sigma}^{(0)} = \boldsymbol{\sigma}^{(1)} = 1$ *and* $\boldsymbol{\gamma}^{(0)} = \boldsymbol{\gamma}^{(1)} = 0$, $\chi^{(0)} \in [0.3, 0.4]$ *and* $\chi^{(1)} \in [0.4, 0.55]$ *(pairwise). The black line corresponds to the true values.*

# Necessary gain from univariate to bivariate analysis (equal dependence but different weights)

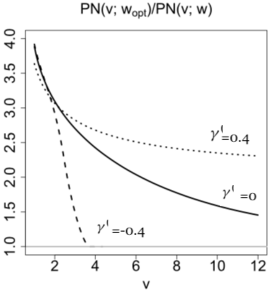



FIG. 5. *Necessary causation gain for* $\boldsymbol{X}^{(0)} \overset{\mathrm{d}}{=} \boldsymbol{Z}^{(0)} \sim \mathrm{MGPD}(\boldsymbol{T}^{(0)}, (1,2)^T, \gamma \mathbf{1}_d)$ *and* $\boldsymbol{X}^{(1)} \overset{\mathrm{d}}{=} \boldsymbol{Z}^{(1)} \sim \mathrm{MGPD}(\boldsymbol{T}^{(1)}, (1.5,2)^T, \gamma \mathbf{1}_d)$, *where* $\boldsymbol{T}^{(1)}$, $\boldsymbol{T}^{(0)}$ *are Gaussian random vectors such that* $\chi^{(0)} = \chi^{(1)} = 0.5$. *The ratio of* $\mathrm{PN}(v, (w_{\mathrm{opt}}, 1 - w_{\mathrm{opt}})^T)$ *to* $\mathrm{PN}(v, (0.5, 0.5)^T)$ *is shown as a function of* $v$, *where* $w_{\mathrm{opt}} = 1$ *based on Proposition* 4.1. *The dashed, solid and dotted lines correspond to a shape parameter (with constraint* $\gamma^{(0)} = \gamma^{(1)}$) *of* $-0.4$, $0$ *and* $0.4$, *respectively.*

# Climate model (CNRM)

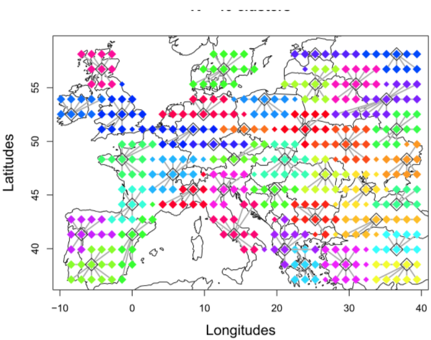
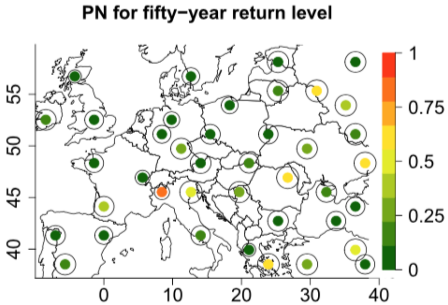



FIG. 7. *Clustering of weekly maximum winter precipitation in central Europe between January* 1985 *and August* 2014, *using the PAM algorithm with distance based on pairwise tail dependence coefficients.*

# Weights influence



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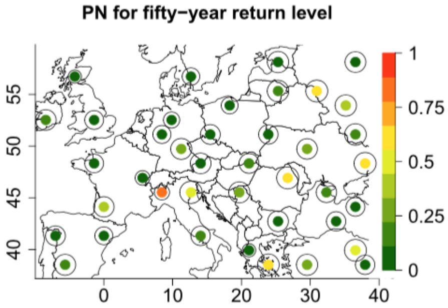


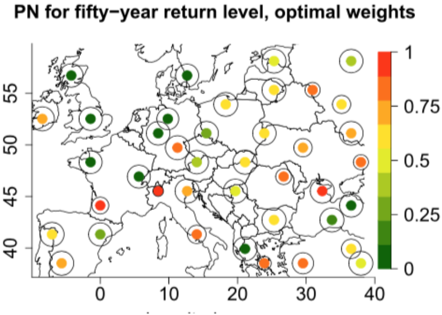

### Projection $\mathbf{w}^T\mathbf{V}^{(i)}$ with $\mathbf{V}_{\gamma=0} = E + \mathbf{T} - \max_{1\le j\le d} T_j$

$$\mathbb{P}[\mathbf{w}^T\mathbf{V} > v] = \frac{1}{(\gamma v + \mathbf{w}^T\sigma)^{1/\gamma}}\mathbb{E}\left[\left(\sum_{j=1}^{d} w_j\sigma_j e^{\gamma(T_j - \max(\mathbf{T}))}\right)^{1/\gamma}\right] \text{ if } \gamma \neq 0,$$

## Projection $\mathbf{w}^T\mathbf{V}^{(i)}$ with $\mathbf{V}_{\gamma=0} = E + \mathbf{T} - \max_{1 \leq j \leq d} T_j$ with $\gamma = 1$

If $\mathbf{T}$ a bivariate normal r.v. with $\mathrm{Var}(T_i) = 1$ & $\mathrm{Cor}(T_1, T_2) = \rho$, then

$$\mathbb{P}[\mathbf{w}^T\mathbf{V} > v] = \frac{\mathbf{w}^T\sigma}{v + \mathbf{w}^T\sigma} \left\{ e^{1-\rho} \Phi \left( -\sqrt{2(1-\rho)} \right) + \frac{1}{2} \right\}.$$

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## PNS

$$PNS(v, \mathbf{w}) = c_1\frac{\mathbf{w}^T\sigma^{(\mathbf{1})}}{v + \mathbf{w}^T\sigma^{(\mathbf{1})}} - c_0\frac{\mathbf{w}^T\sigma^{(\mathbf{0})}}{v + \mathbf{w}^T\sigma^{(\mathbf{0})}}$$

with $c_i = e^{1-\rho^{(i)}}\Phi\left(-\sqrt{2(1-\rho^{(i)})}\right) + \frac{1}{2}$

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with $c_i = e^{1-\rho^{(i)}}\Phi\left(-\sqrt{2(1-\rho^{(i)})}\right) + \frac{1}{2}$

## Maximizing PNS with respect to $v$

$$v^* = \frac{(\mathbf{w}^T\sigma^{(0)})(\mathbf{w}^T\sigma^{(1)}) + \sqrt{(\mathbf{w}^T\sigma^{(0)})(\mathbf{w}^T\sigma^{(1)})c_0c_1}}{c_1(\mathbf{w}^T\sigma^{(1)}) - c_0(\mathbf{w}^T\sigma^{(0)})}$$

Also explicit expressions for maximizing PNS with respect to **w**, but particularly ugly

## Example : $\gamma = 1$ and $\mathbf{V}_{\gamma=0} = E + \mathbf{T} - \max_{1 \le j \le d} T_j$ with T a bivariate normal

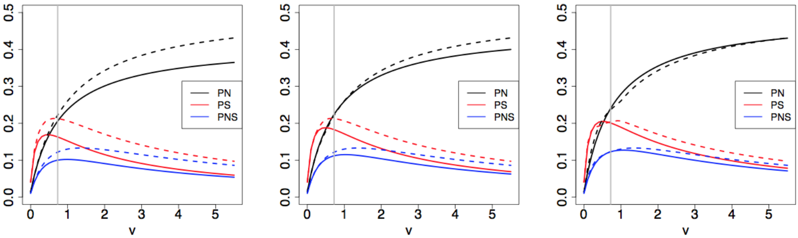



Figure 4: PN, PS and PNS for $\gamma = 1$ and $\boldsymbol{\sigma}^{(0)} = (1, 0.6)$, for $\boldsymbol{\sigma}^{(1)} = (2, 0.7)$ (left), $\boldsymbol{\sigma}^{(1)} = (2, 0.9)$ (middle) and $\boldsymbol{\sigma}^{(1)} = (2, 1.1)$ (right), for $w = 1/2$ (solid lines) and $w = w^*_{\mathrm{PNS}}$ (dashed lines). The dependence structure is as in Example 2.2 with a correlation of 0.3 in the factual world and 0.4 in the counterfactual world.

## Example : $\gamma = 0$ and $\mathbf{V}_{\gamma=0} = E + \mathbf{T} - \max_{1 \leq j \leq d} T_j$ with $\mathbf{T}$ a bivariate normal

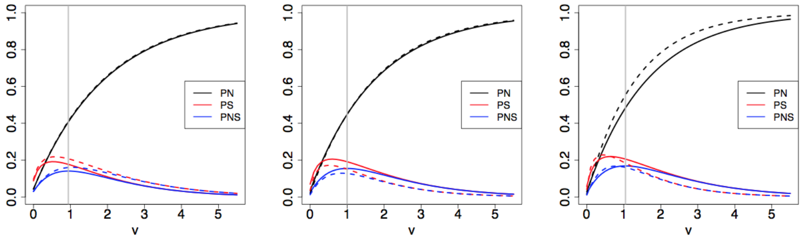



Figure 3: PN, PS and PNS for $\gamma = 0$ and $\boldsymbol{\sigma}^{(0)} = (1, 0.6)$, for $\boldsymbol{\sigma}^{(1)} = (2, 0.7)$ (left), $\boldsymbol{\sigma}^{(1)} = (2, 0.9)$ (middle) and $\boldsymbol{\sigma}^{(1)} = (2, 1.1)$ (right), for $w = 1/2$ (solid lines) and $w = w^*_{\text{PN}}$ (dashed lines). The dependence structure is as in Example 2.2 with a correlation of 0.3 in the factual world and 0.4 in the counterfactual world.

# Conclusions

## Main messages

- Interesting links between Extreme Value Theory and Causality Theory in attributing climate extremes
- Multivariate Pareto projections can be optimized with respect to causation criteria
- In climate studies, this can help contrasting the impact of anthropogenic effect

## Future work on difference sources of error and/or uncertainty in D&A

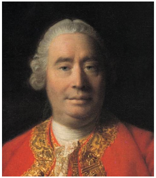
- Natural climate internal variability
- Natural forcing variations
- Model uncertainty from approximating the true climate system with numerical experiments
- Observational uncertainties due to instrumental errors, homogenization problems and mismatches between data sources
- Sampling uncertainty in space and time
- Statistical modeling error by assuming a specific statistical model, e.g., assuming a generalized extreme value distribution for independent block maxima.
- Inferential uncertainties

PhD ad on "Development of machine learning methods to combine multi-model biases in studies of detection and attribution of climatic extremes" (contact me).

## The cornerstone of causality: counterfactual definition

- D. Hume, *An Enquiry Concerning Human Understanding*,1748

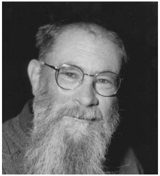
  *« We may define a cause to be an object followed by another, where, if the first object had not been, the second never had existed. »*



*D. Hume, 18th century*

- D. K. Lewis, *Counterfactuals*, 1973

  *« We think of a cause as something that makes a difference, and the difference it makes must be a difference from what would have happened without it. Had it been absent, its effects would have been absent as well. »*

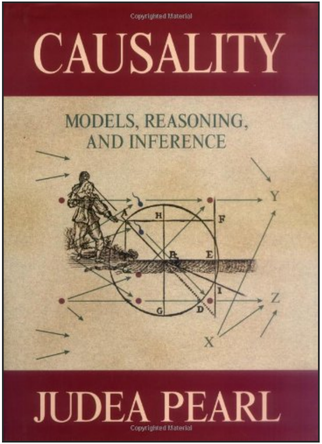


*D. Lewis, 20th century*

see, e.g. Hannart, A., Pearl J. Otto F., P. Naveau and M. Ghil. (BAMS, 2015). Counterfactual causality theory for the attribution of weather and climate-related events

# Consolidation of a standard causality theory (1980-1990)

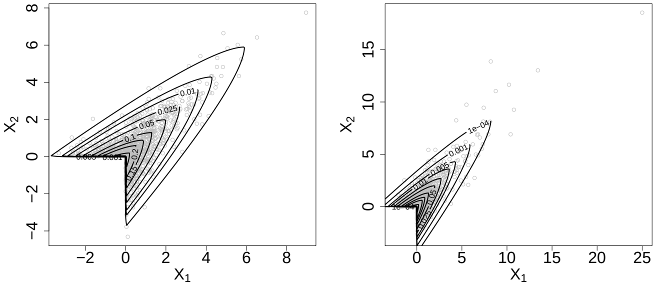
- Common theoretical corpus on causality
  - what does «X causes Y» mean ?
  - how does one evidence a causality link from data ?
  - philosophy, artificial intelligence, statistics.
  - statistics alone not enough - more concepts needed.




- J. Pearl (2000), *Causality: models, reasoning and inference*, Cambridge University Press.

- Turing Award 2004.

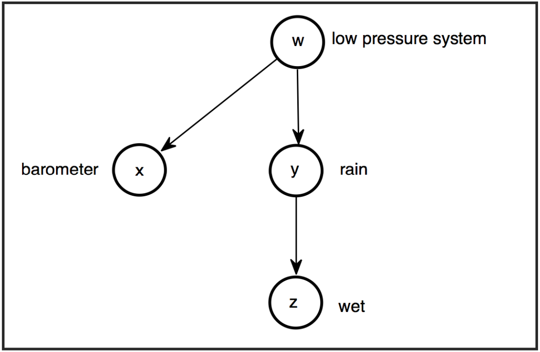
- Provides clear semantics and sound logic for causal reasoning.




Cor($T_1$ , $T_2$) $= 0.4$, with $\gamma = \mathbf{0}$ (left) and $\gamma = (0.2, 0.2)$ (right)

# Oriented graphs

- visual representation of the conditional independence structure of a joint distribution




$$P(X,Y,Z,W) = P(W)\,.\,P(X \mid W)\,.\,P(Y \mid W)\,.\,P(Z \mid Y)$$

# Interventional probability

- Limitation of oriented graphs
  - identifiability: several causal graphs are compatible with the same pdf (and hence with the same observations).

$$P(X,Y) = \underbrace{P(X)\,.\,P(Y \mid X)}_{X \,\to\, Y} = \underbrace{P(Y)\,.\,P(X \mid Y)}_{Y \,\to\, X}$$

  - Need for disambiguation.

$\Rightarrow$ **<u>experimentation</u>**

# Interventional probability

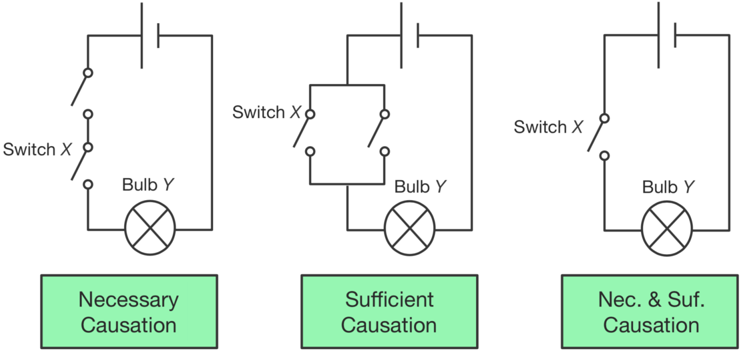
- New notion:
  - intervention $do(X=x)$
  - interventional probability $P(Y \mid do(X=x)) = P(Y_x)$

the probability of rain **forcing** the barometer to decrease,
in an experimental context in which the barometer is manipulated

$$P(Y \mid do(X=x)) \neq P(Y \mid X=x)$$

the probability of rain **knowing** that the barometer is decreasing,
in a non-experimental context in which the barometer evolution is left unconstrained

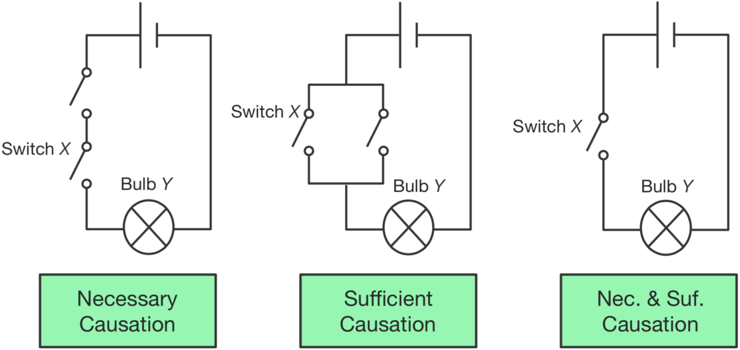
# Causal theory (Pearl): causality has two facets




Source : A. Hannart & PN, Journal of Climate, 2018

# Causal theory (Pearl): Probabilities of causation

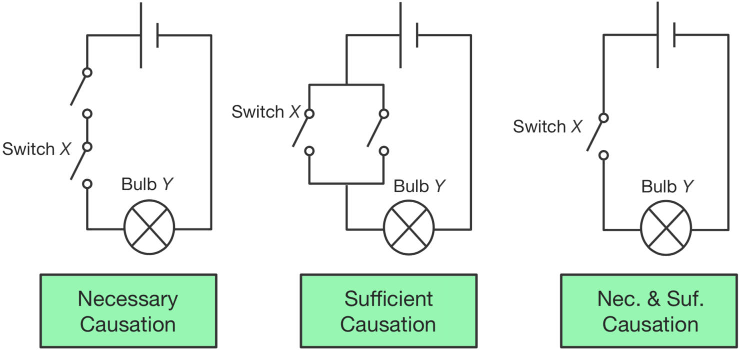
- **Probability of necessary causation** = probability that the effect is removed when the cause is turned off, conditional on the fact that the effect and the cause were initially present.




Source : A. Hannart & PN, Journal of Climate, 2018

# Causal theory (Pearl): Probabilities of causation

- **Probability of sufficient causation** = probability that the effect appears when the cause is turned on, conditional on the fact that the effect and the cause were initially absent.




Source : A. Hannart & PN, Journal of Climate, 2018

## Fundamental difference : necessary and sufficient causation

- Definitions:
  - "*X is a necessary cause of Y*" means that X is required for Y to occur but that other factors might be required as well.
  - "*X is a sufficient cause of Y*" means that X always triggers Y but that Y may also occur for other reasons without requiring X.
- Examples:
  - clouds are a necessary cause of rain but not a sufficient one.
  - rain is a sufficient cause for the road being wet, but not a necessary one.

## Fundamental difference : necessary and sufficient causation

- Definitions:
  - **Probability of necessary causality = PN** = the probability that the event Y would not have occurred in the absence of the event X given that both events Y and X did in fact occur.
  - **Probability of sufficient causation = PS** = the probability that Y would have occurred in the presence of X, given that Y and X did not occur.
- Formalization:

$$\begin{cases} \mathrm{PN} =_{\text{def}} P(Y_0 = 0 \mid Y = 1, X = 1) \\ \mathrm{PS} =_{\text{def}} P(Y_1 = 1 \mid Y = 0, X = 0) \\ \mathrm{PNS} =_{\text{def}} P(Y_0 = 0, Y_1 = 1) \end{cases}$$